EXPONENTIALS AND LOGARITHMS

1 Express in the form $p \log_{10} a + q \log_{10} b$

a $\log_{10} ab$ **b** $\log_{10} ab^7$ **c** $\log_{10} \frac{a^3}{b}$ **d** $\log_{10} a\sqrt{b}$

e $\log_{10} (ab)^2$ **f** $\log_{10} \frac{1}{ab}$ **g** $\log_{10} \sqrt{a^3 b^5}$ **h** $3 \log_{10} \frac{a^2}{\sqrt[3]{b}}$

2 Given that $y = \log_q 8$, express each of the following in terms of y.

a $\log_q 64$ **b** $\log_q 2$ **c** $\log_q \frac{16}{q}$ **d** $\log_q 4q^3$

3 Given that $a = \lg 2$ and $b = \lg 3$, express each of the following in terms of a and b.

a $\lg 18$ **b** $\lg 96$ **c** $\lg \frac{9}{16}$ **d** $\lg 6 - \lg 8$

e $\lg \sqrt{6}$ **f** $\frac{3}{2} \lg 16 + \frac{1}{2} \lg 81$ **g** $4 \lg 3 - 3 \lg 6$ **h** $\lg 60 + \lg 20 - 2$

4 Without using a calculator, evaluate

a $\frac{1}{3} \log_5 1000 - \frac{1}{2} \log_5 4$ **b** $2 \log_{12} 4 + \frac{1}{2} \log_{12} 81$ **c** $\log_4 12 + \log_4 \frac{2}{3}$

d $\frac{\log_7 81}{\log_7 3}$ **e** $3 \log_{27} 12 - 2 \log_{27} 72$ **f** $\frac{\log_{11} 25}{\log_{11} \frac{1}{5}}$

5 Solve each equation, giving your answers correct to 3 significant figures.

a $\log_3 x = 1.8$ **b** $\log_5 x = -0.3$ **c** $\log_8 (x - 3) = 2.1$

d $\log_4 (\frac{1}{2}x + 1) = 3.2$ **e** $15 - \log_2 3y = 9.7$ **f** $\log_6 (1 - 5t) + 4.2 = 3.6$

6 Express in the form $\log_2 [f(x)]$

a $5 \log_2 x$ **b** $\log_2 x + \log_2 (x + 4)$ **c** $2 \log_2 x + \frac{1}{5} \log_2 x^5$

d $3 \log_2 (x - 2) - 4 \log_2 x$ **e** $\log_2 (x^2 - 1) - \log_2 (x + 1)$ **f** $\log_2 x - \frac{1}{2} \log_2 x^4 + \frac{1}{3} \log_2 x^2$

7 Solve each of the following equations.

a $\log_3 x + \log_3 5 = \log_3 (2x + 3)$ **b** $\log_9 x + \log_9 10 = \frac{3}{2}$

c $\log_4 x - \log_4 (x - 1) = \log_4 3 + \frac{1}{2}$ **d** $\log_5 5x - \log_5 (x + 2) = \log_5 (x + 6) - \log_5 x$

e $2 \log_6 x = \log_6 (2x - 5) + \log_6 5$ **f** $\log_7 4x = \log_7 \frac{1}{x - 6} + 1$

8 Solve each pair of simultaneous equations.

a $\log_x y = 2$
$xy = 27$

b $\log_5 x - 2 \log_5 y = \log_5 2$
$x + y^2 = 12$

c $\log_2 x = 3 - 2 \log_2 y$
$\log_y 32 = -\frac{5}{2}$

d $\log_y x = \frac{3}{2}$
$x^{\frac{1}{3}} + 3y^{\frac{1}{2}} = 20$

e $\log_a x + \log_a 3 = \frac{1}{2} \log_a y$
$3x + y = 20$

f $\log_{10} y + 2 \log_{10} x = 3$
$\log_2 y - \log_2 x = 3$